Relations & Functions

1. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$. Determine
   (i) $A \times B$ (ii) $B \times A$
   (iii) $B \times B$ (iv) $A \times A$
2. If $P = \{x : x < 3, x \in \mathbf{N}\}$, $Q = \{x : x \leq 2, x \in \mathbf{W}\}$. Find $(P \cup Q) \times (P \cap Q)$, where $\mathbf{W}$ is the set of whole numbers.
3. If $A = \{x : x \in \mathbf{W}, x < 2\}$ $B = \{x : x \in \mathbf{N}, 1 < x < 5\}$ $C = \{3, 5\}$ find
   (i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$
4. In each of the following cases, find a and b.
   (i) $(2a + b, a - b) = (8, 3)$ (ii) $\left(\frac{a}{4}, a-2b\right) = (0, 6 + b)$
5. Given $A = \{1, 2, 3, 4, 5\}$, $S = \{(x, y) : x \in A, y \in A\}$. Find the ordered pairs which satisfy the conditions given below:
   (i) $x + y = 5$ (ii) $x + y < 5$ (iii) $x + y > 8$
6. Given $R = \{(x, y) : x, y \in \mathbf{W}, x^2 + y^2 = 25\}$. Find the domain and Range of R.
7. If $R_1 = \{(x, y) \mid y = 2x + 7$, where $x \in \mathbf{R}$ and $-5 \leq x \leq 5\}$ is a relation. Then find the domain and Range of R_1.
8. If $R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$ is a relation. Then find R_2.
9. If $R_3 = \{(x, |x|) \mid x$ is a real number$\}$ is a relation. Then find domain and range of R_3.
10. Is the given relation a function? Give reasons for your answer.
    (i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$
    (ii) $f = \{(x, x) \mid x$ is a real number$\}$
    (iii) $g = \left\{\left(n, \frac{1}{n}\right) \mid n \text{ is a positive integer}\right\}$
    (iv) $s = \{(n, n^2) \mid n$ is a positive integer$\}$
    (v) $t = \{(x, 3) \mid x$ is a real number.
11. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following
    (a) $f(3) + g(-5)$ (b) $f\left(\frac{1}{2}\right) \times g(14)$

(c) $f(-2)+g(-1)$ (d) $f(t)-f(-2)$

(e) $\frac{f(t)-f(5)}{t-5}$, if $t \neq 5$

12. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.
    (a) For what real numbers x, $f(x) = g(x)$?
    (b) For what real numbers x, $f(x) < g(x)$?
13. If f and g are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$, then find.

    (i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$

14. Express the following functions as set of ordered pairs and determine their range.
    $f: X \to R, f(x) = x^3 + 1$, where $X = \{-1, 0, 3, 9, 7\}$
15. Find the values of x for which the functions
    $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal
16. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?
17. Find the domain of each of the following functions given by

    (i) $f(x) = \frac{1}{\sqrt{1-\cos x}}$ (ii) $f(x) = \frac{1}{\sqrt{x+|x|}}$

    (iii) $f(x) = x\,|x|$ (iv) $f(x) = \frac{x^3 - x + 3}{x^2 - 1}$

    (v) $f(x) = \frac{3x}{2x-8}$

18. Find the range of the following functions given by

(i) $f(x) = \frac{3}{2-x^2}$ (ii) $f(x) = 1 - |x-2|$

(iii) $f(x) = |x-3|$ (iv) $f(x) = 1 + 3\cos 2x$

(Hint : $-1 \leq \cos 2x \leq 1 \Rightarrow -3 \leq 3\cos 2x \leq 3 \Rightarrow -2 \leq 1 + 3\cos 2x \leq 4$)

19. Redefine the function $f(x) = |x-2| + |2+x|$. $-3 \leq x \leq 3$

20. If $f(x) = \frac{x-1}{x+1}$, then show that

(i) $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

21. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$. Find

(i) $(f+g)(x)$ (ii) $(f-g)(x)$

(iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

22. Find the domain and Range of the function $f(x) = \frac{1}{\sqrt{x-5}}$.

23. If $f(x) = y = \frac{ax-b}{cx-a}$, then prove that $f(y) = x$.

Choose the correct answers in Exercises from 24 to 35 (M.C.Q.)

24. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) $n^m - 1$

(C) $mn - 1$ (D) $2^{mn} - 1$

25. If $[x]^2 - 5[x] + 6 = 0$, where $[\ .\ ]$ denote the greatest integer function, then

(A) $x \in [3, 4]$ (B) $x \in (2, 3]$

(C) $x \in [2, 3]$ (D) $x \in [2, 4)$

26. Range of $f(x) = \dfrac{1}{1-2\cos x}$ is

(A) $\left[\frac{1}{3}, 1\right]$ (B) $\left[-1, \frac{1}{3}\right]$

(C) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$ (D) $\left[-\frac{1}{3}, 1\right]$

27. Let $f(x) = \sqrt{1+x^2}$, then

(A) $f(xy) = f(x) \cdot f(y)$ (B) $f(xy) \geq f(x) \cdot f(y)$

(C) $f(xy) \leq f(x) \cdot f(y)$ (D) None of these

[Hint : find $f(xy) = \sqrt{1+x^2y^2}$, $f(x) \cdot f(y) = \sqrt{1+x^2y^2+x^2+y^2}$]

28. Domain of $\sqrt{a^2-x^2}$ $(a > 0)$ is

(A) $(-a, a)$ (B) $[-a, a]$

(C) $[0, a]$ (D) $(-a, 0]$

29. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to

(A) $a = -3, b = -1$ (B) $a = 2, b = -3$

(C) $a = 0, b = 2$ (D) $a = 2, b = 3$

30. The domain of the function f defined by $f(x) = \sqrt{4-x} + \dfrac{1}{\sqrt{x^2-1}}$ is equal to

(A) $(-\infty, -1) \cup (1, 4]$ (B) $(-\infty, -1] \cup (1, 4]$

(C) $(-\infty, -1) \cup [1, 4]$ (D) $(-\infty, -1) \cup [1, 4)$

31. The domain and range of the real function f defined by $f(x) = \dfrac{4-x}{x-4}$ is given by

(A) Domain = R, Range = $\{-1, 1\}$

(B) Domain = $\mathbf{R} - \{1\}$, Range = $\mathbf{R}$

(C) Domain = $\mathbf{R} - \{4\}$, Range = $\{-1\}$

(D) Domain = $\mathbf{R} - \{-4\}$, Range = $\{-1, 1\}$

32. The domain and range of real function f defined by $f(x) = \sqrt{x-1}$ is given by

(A) Domain = $(1, \infty)$, Range = $(0, \infty)$

(B) Domain = $[1, \infty)$, Range = $(0, \infty)$

(C) Domain = $[1, \infty)$, Range = $[0, \infty)$

(D) Domain = $[1, \infty)$, Range = $[0, \infty)$

33. The domain of the function f given by $f(x) = \dfrac{x^2+2x+1}{x^2-x-6}$

(A) $\mathbf{R} - \{3, -2\}$ (B) $\mathbf{R} - \{-3, 2\}$

(C) $\mathbf{R} - [3, -2]$ (D) $\mathbf{R} - (3, -2)$

34. The domain and range of the function f given by $f(x) = 2 - |x-5|$ is

(A) Domain = $\mathbf{R}^+$, Range = $(-\infty, 1]$

(B) Domain = $\mathbf{R}$, Range = $(-\infty, 2]$

(C) Domain = $\mathbf{R}$, Range = $(-\infty, 2)$

(D) Domain = $\mathbf{R}^+$, Range = $(-\infty, 2]$

35. The domain for which the functions defined by $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal is

(A) $\left\{-1, \frac{4}{3}\right\}$ (B) $\left[-1, \frac{4}{3}\right]$

(C) $\left(-1, \frac{4}{3}\right)$ (D) $\left[-1, \frac{4}{3}\right)$